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Modelling of eclipsing binaries with pulsating components

Košice 2019

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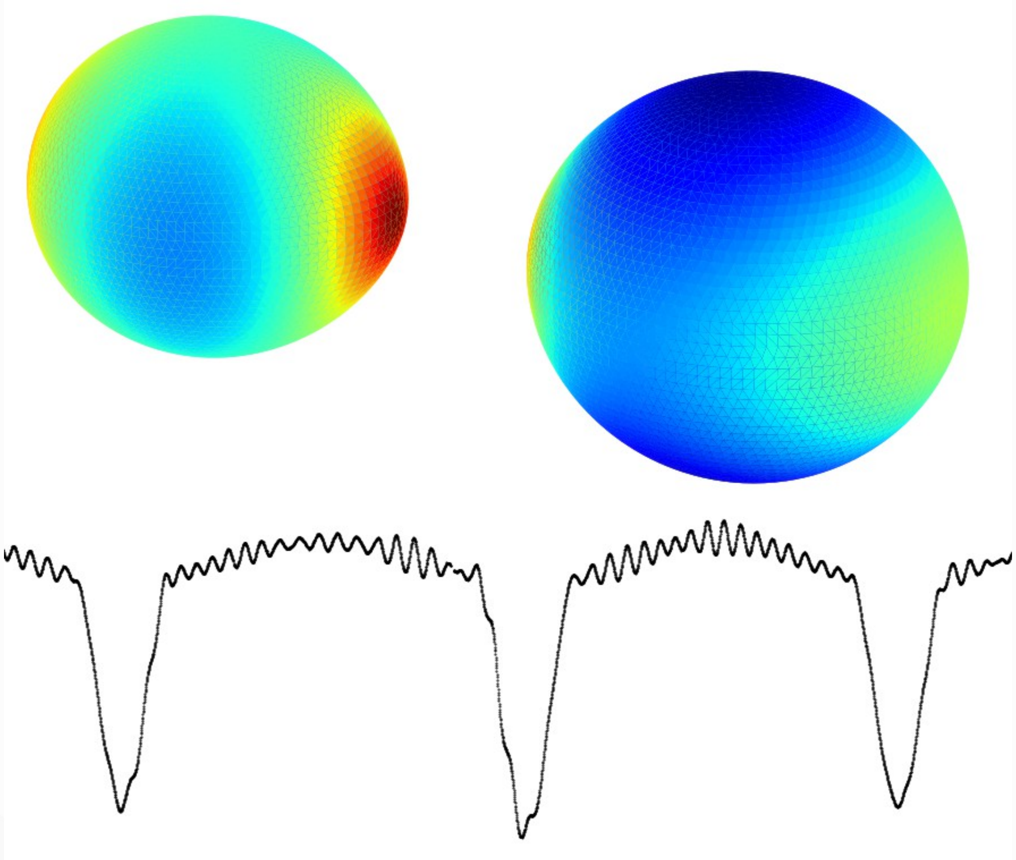
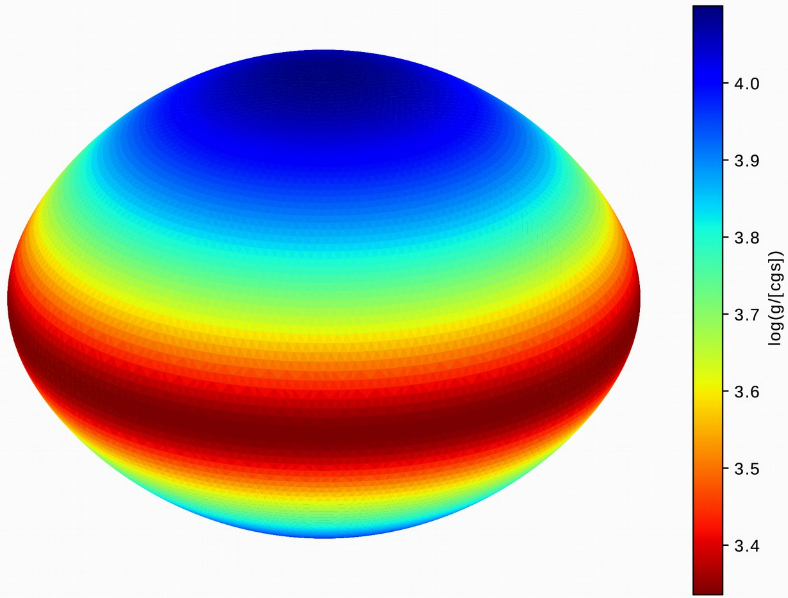
Motivation

- **Large amount of oscillating binaries in data from photometric surveys and space based telescopes.**
- **Eclipsing binaries enable precise derivation of absolute parameters of components.**
- **Great starting point for subsequent asteroseismological analysis.**

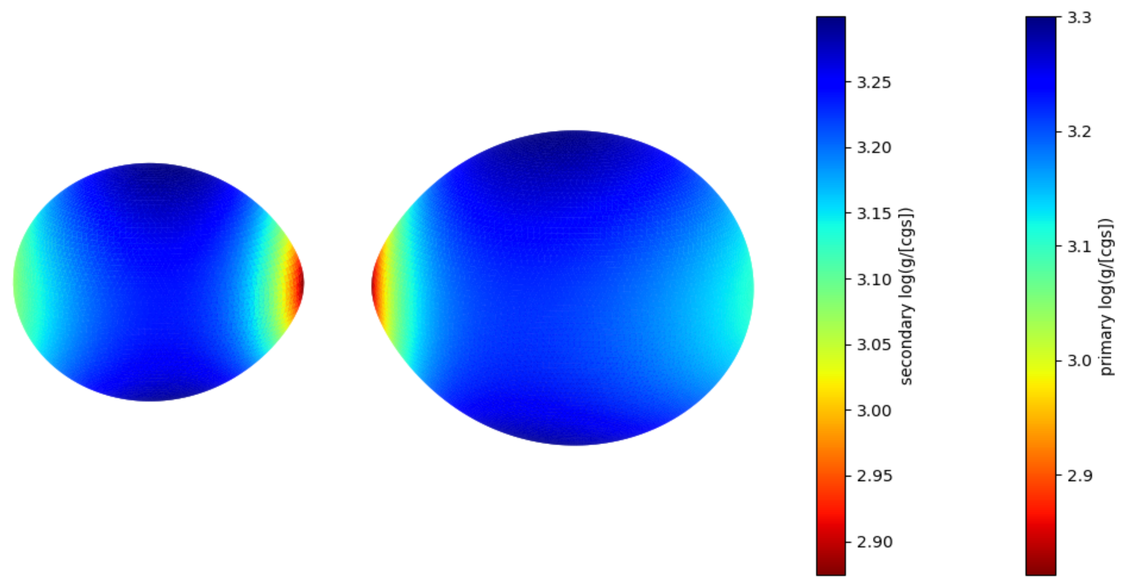


Main objects of interest

- **Rapidly rotating stars**



- **Tidally deformed stars**



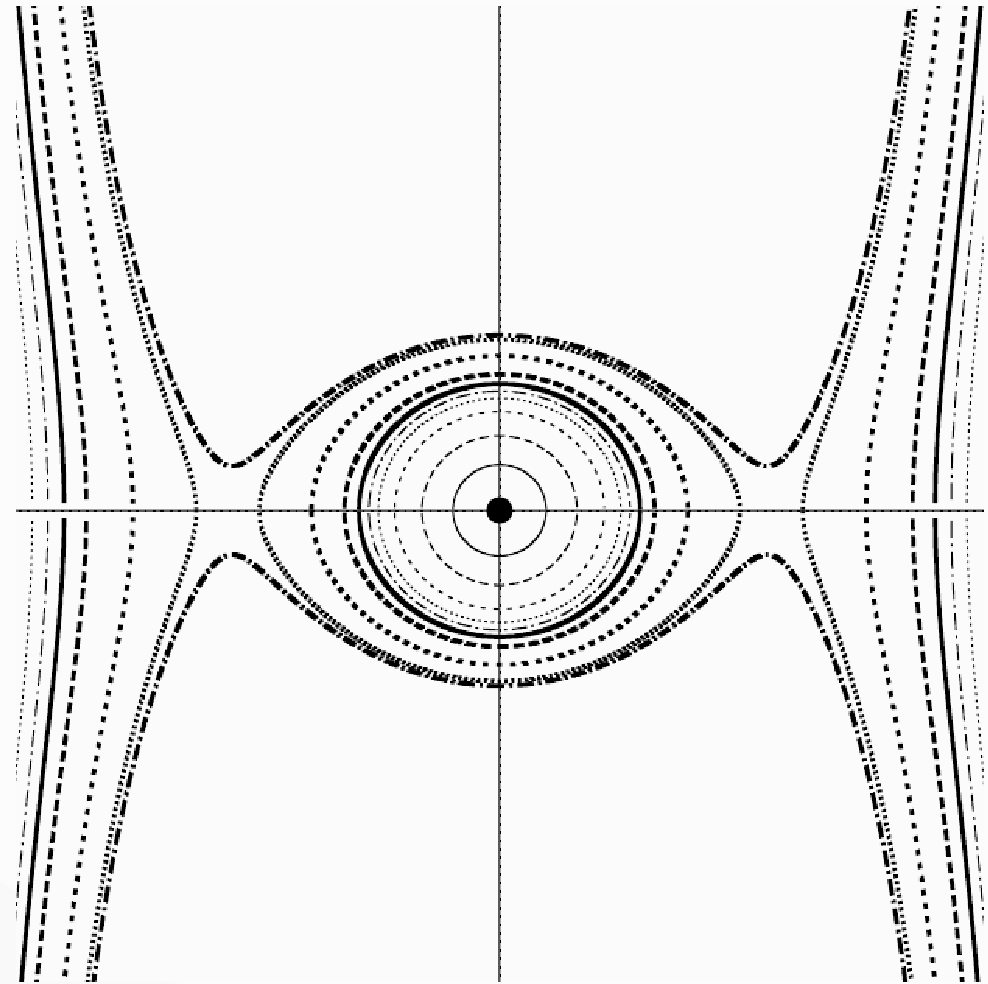
Rapidly rotating stars

- **Combination of gravitational and centrifugal force.**

$$\Psi = G\frac{M}{r} + \frac{1}{2}(r\omega \sin\theta)^2$$

- **Rapid rotation causes equatorial flattening.**
- **Existence of a critical break-up equatorial rotation speed.**

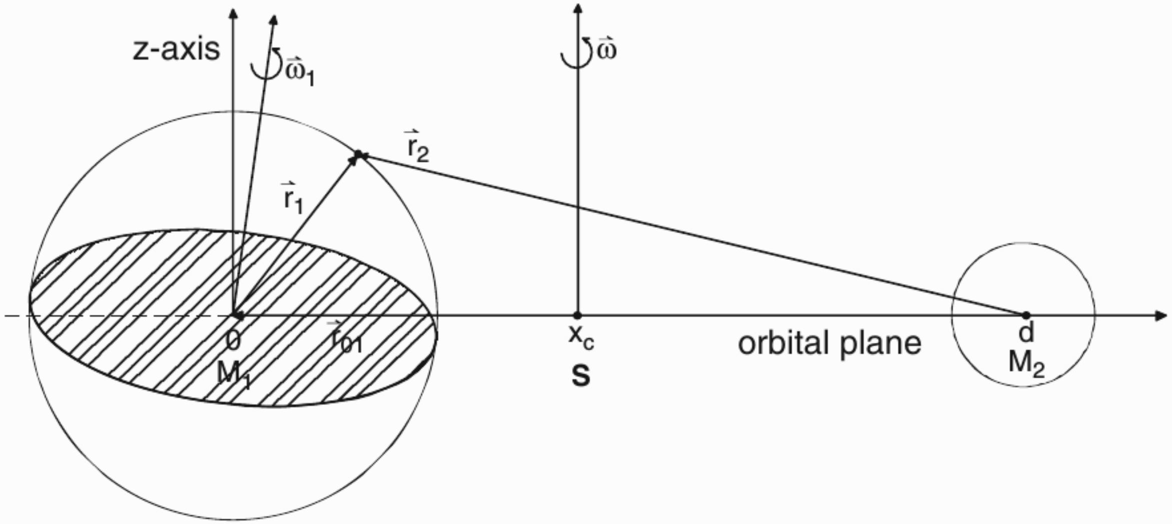
$$v_c = \sqrt[3]{GM\omega}$$



Source: Neslušan (2009)

Roche geometry

- **Restricted 3 body problem**
  - **Stars regarded as a point masses.**
  - **Uniform rotation.**
  - **Surface in hydrostatic equilibrium.**

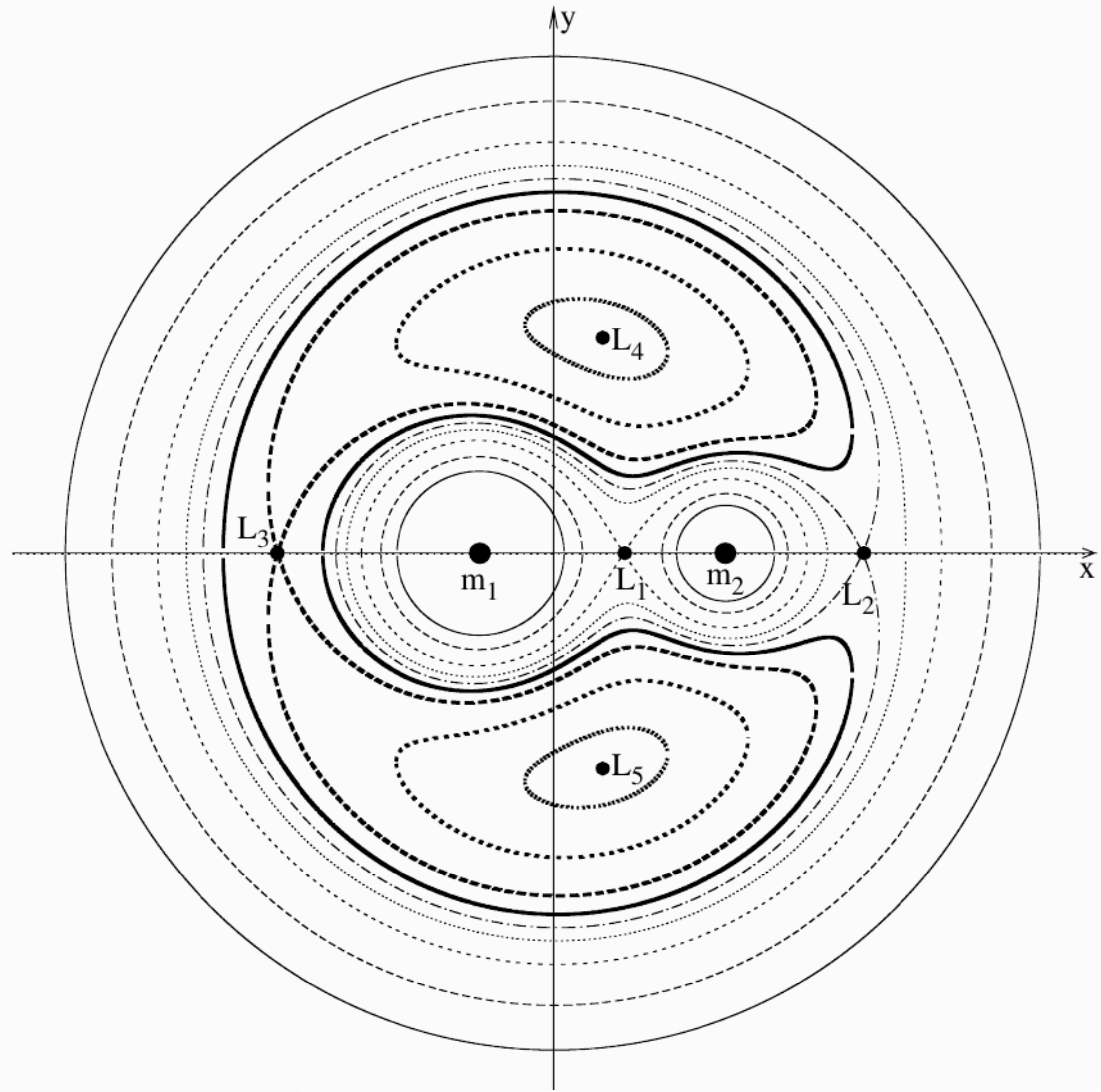


Source: Kallrath & Milone (2009)

$$\Omega(\boldsymbol{\varrho}; q, d) = \frac{1}{\varrho} + q\left[\frac{1}{\sqrt{d^2 - 2d\varrho\cos\varphi\cos\theta + \varrho^2}} + \frac{\varrho\cos\varphi\cos\theta}{d^2}\right] + \frac{1}{2}\left(\frac{\omega_1}{\omega}\right)^2 (q+1)\varrho^2(1-\cos^2\theta)$$

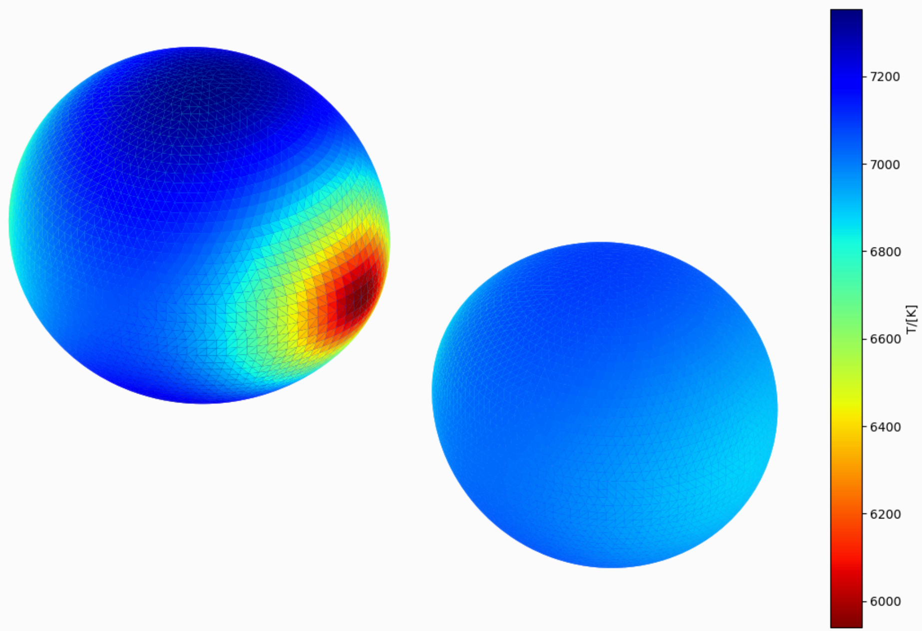
Roche geometry

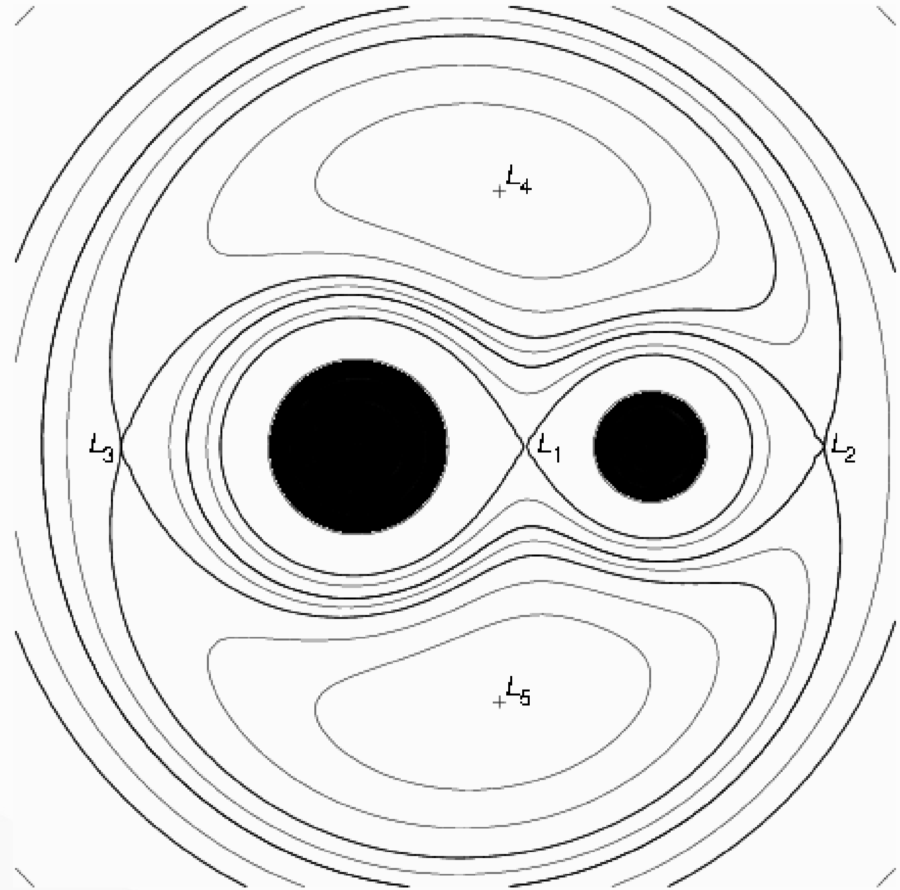
- **Solution: equipotential surfaces.**
- **Solved iteratively.**
- **Tidally deformed surfaces of the components due to the mutual gravitational interaction.**



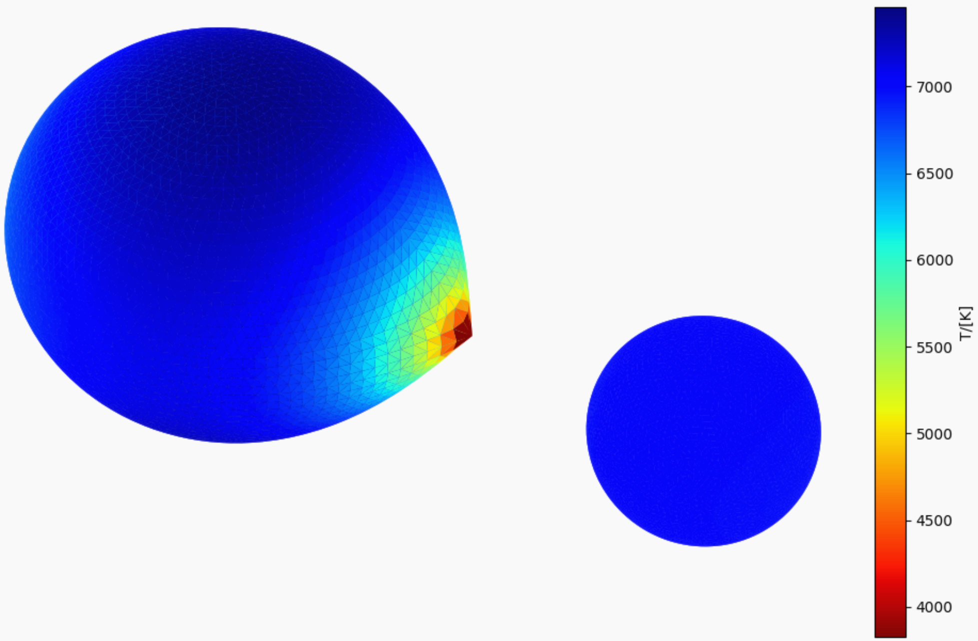
Source: Neslušan (2009)

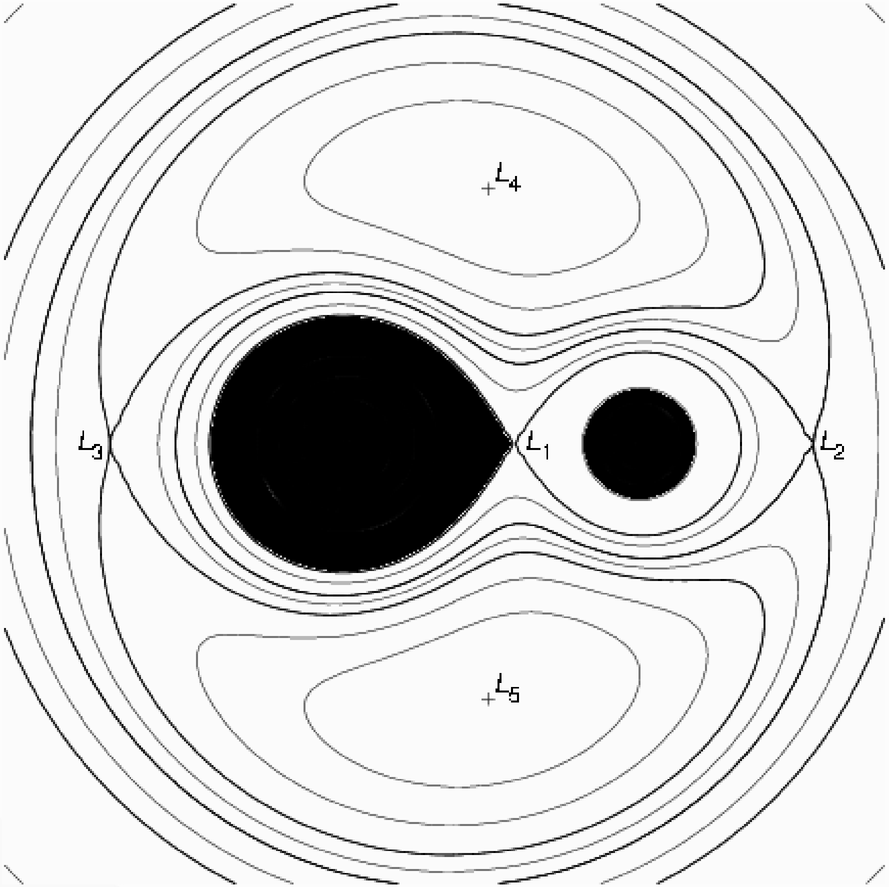
Detached binary



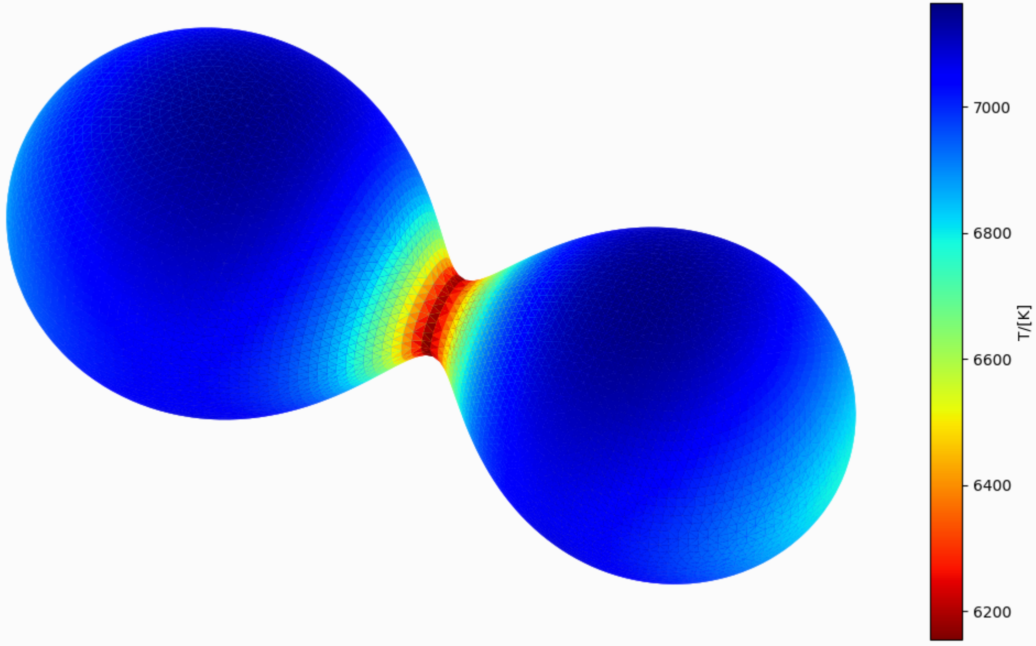


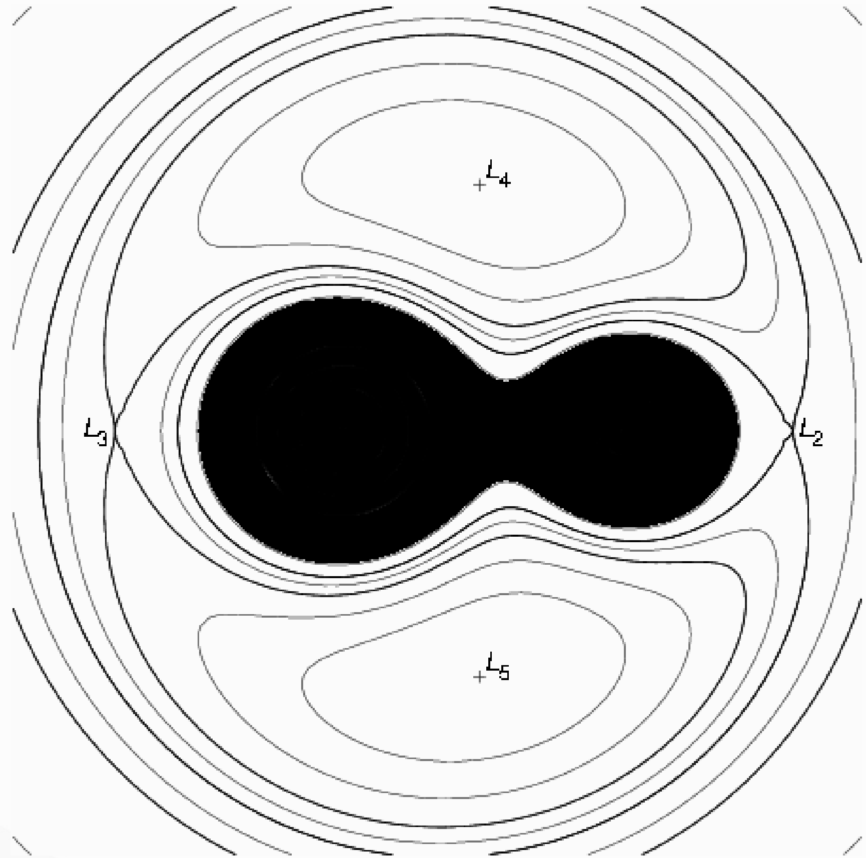
Semi-detached binary



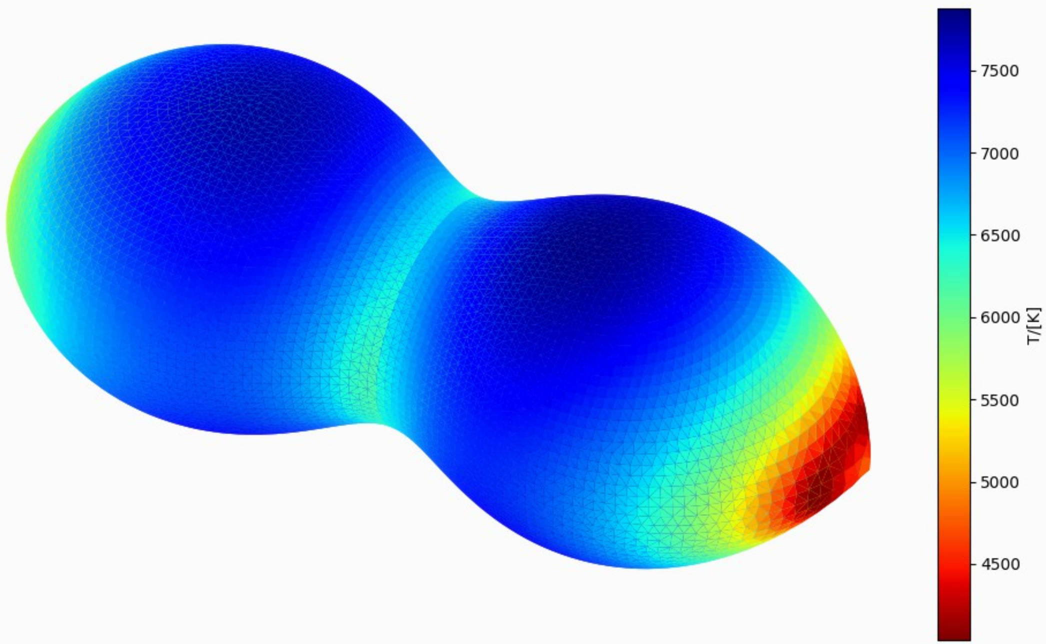


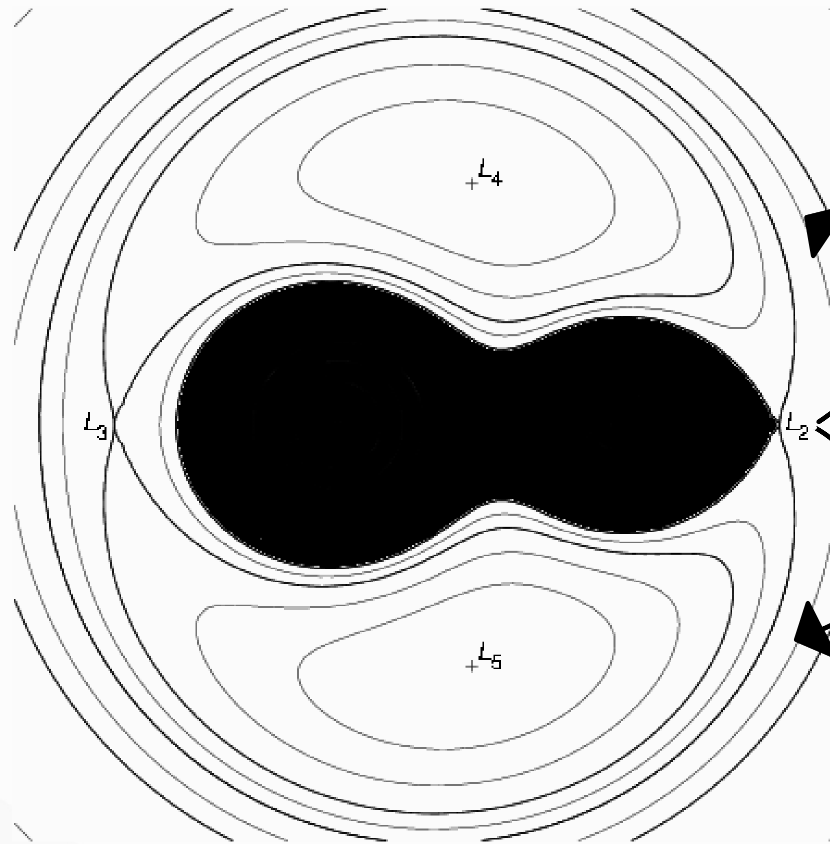
Over-contact binary



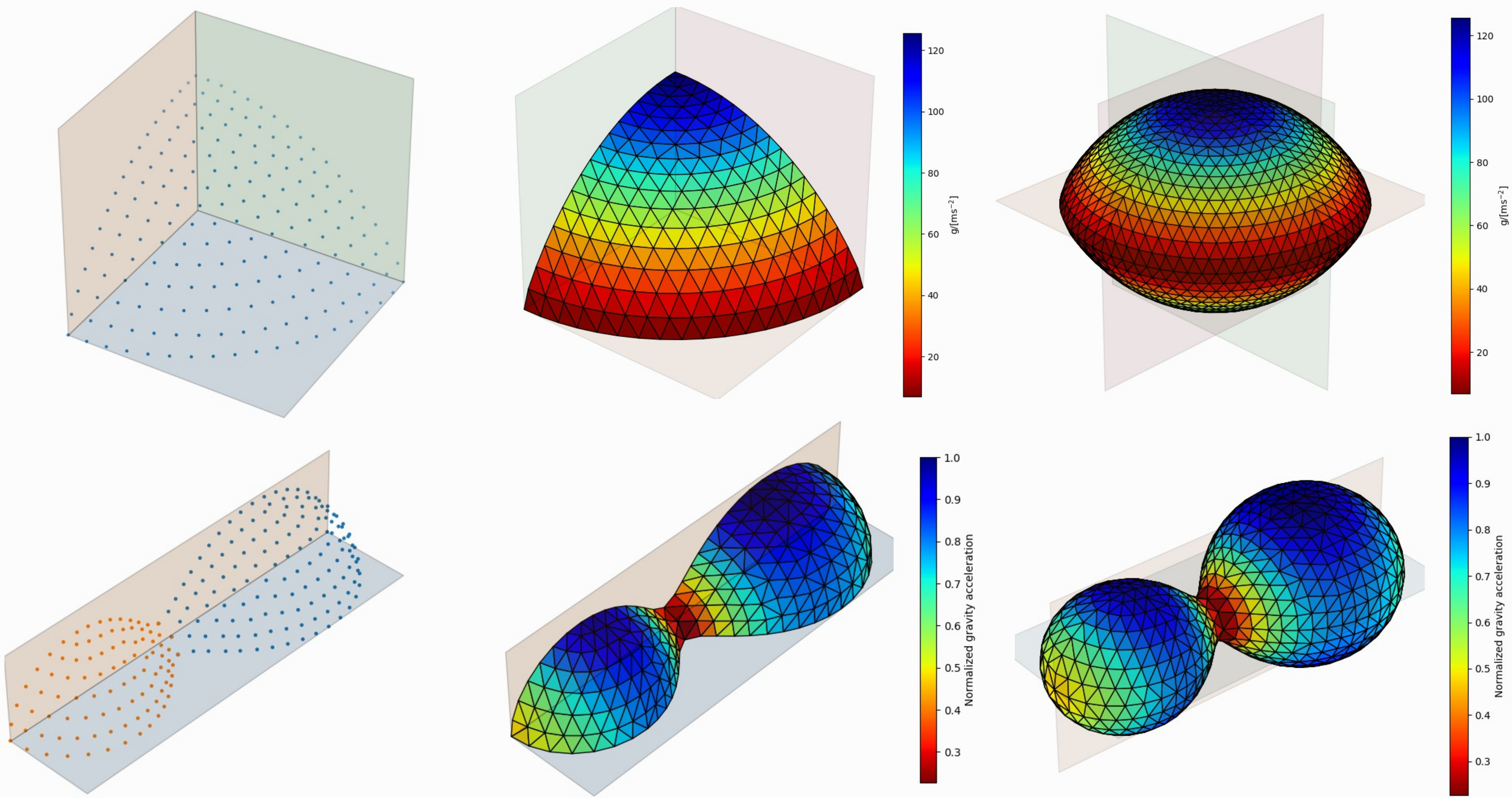


Over-contact binary with loss of material



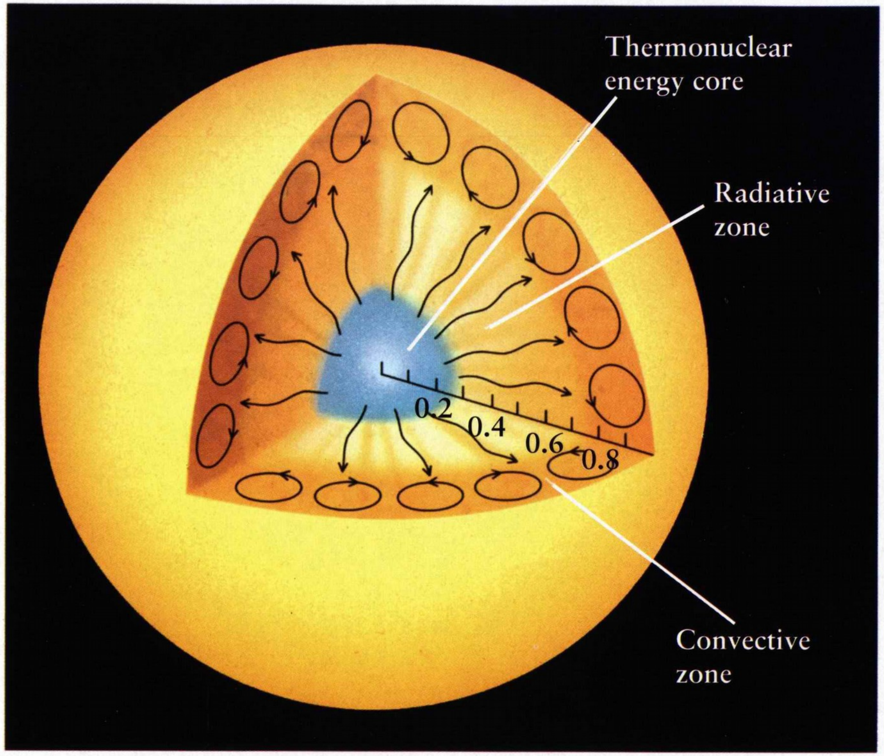


Symmetries of the stellar surfaces



Equilibrium stellar structure

- **Propagation of a stellar oscillations depends heavily on the internal stellar structure.**
- **Research on oscillations - window to internal stellar structure.**



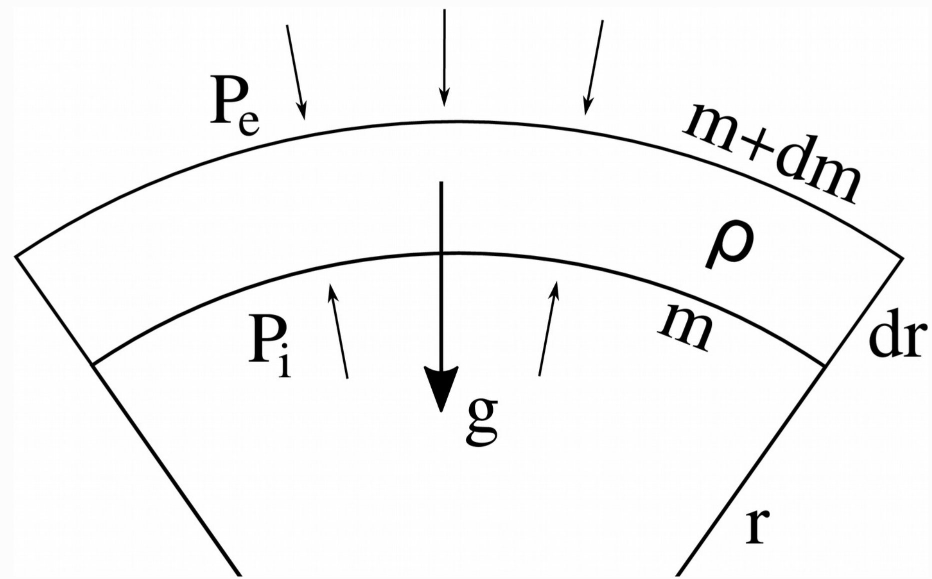
Equilibrium stellar structure

- **Maintained between gravity and internal pressure of material.**

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

- **Switch from independent variable *r* to *m*.**

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$



Equilibrium stellar structure – production of energy

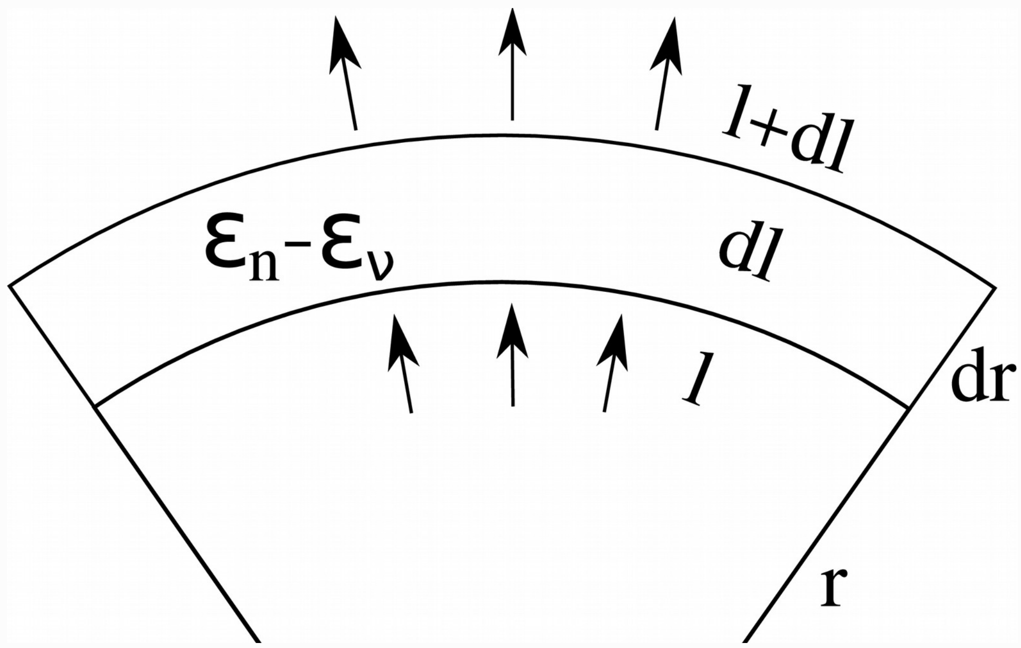
- **Distribution of luminosity *l***

$$\frac{\partial l}{\partial m} = \epsilon_n - \epsilon_\nu$$

- **Energy produced via thermonuclear reactions.**

$$\epsilon = \sum_{ij} \epsilon_{ij} = \frac{1}{\rho} \sum_{ij} r_{ij} e_{ij}$$

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left(\sum_j r_{ji} - \sum_k r_{ik} \right)$$



Equilibrium stellar structure – transport of energy

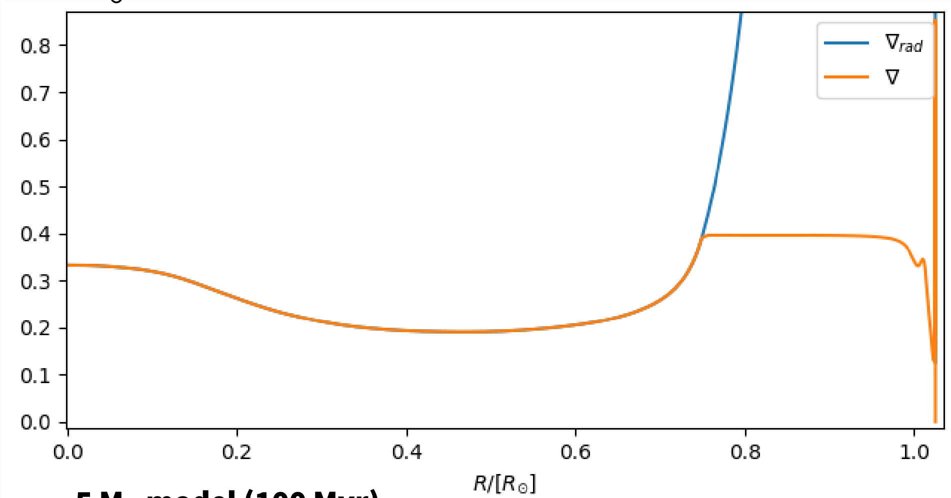
- **Radiation:**
  - **temperature gradient necessary to transport all energy via radiation:**

$$\nabla_{rad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{rad}$$

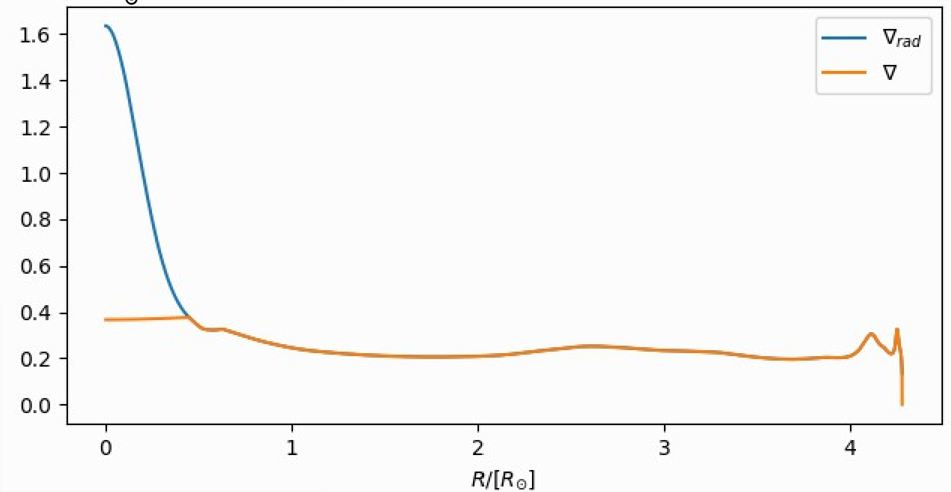
- **Convection:**
  - **Fraction of the energy transported by convection if Ledoux criterion is violated:**

$$\nabla_{rad} < \nabla_{ad} + \frac{\delta}{\varphi}\frac{d\ln \mu}{d\ln P}$$

1 M$_\odot$ model (4.5 Gyr)



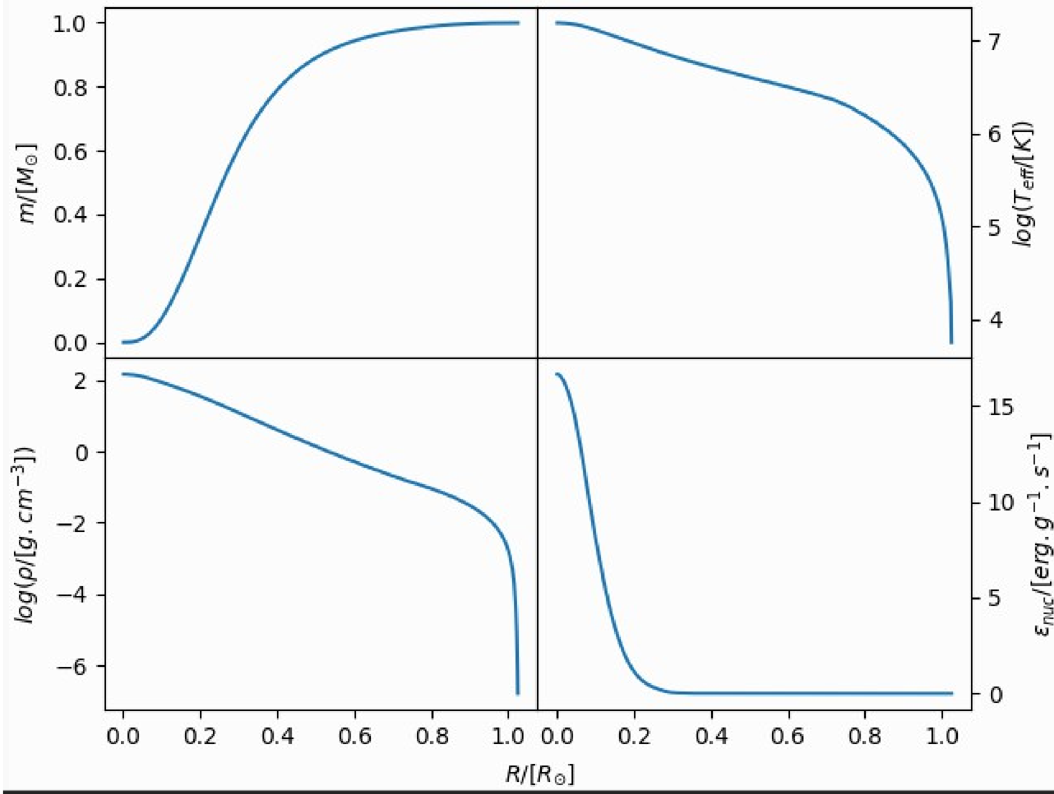
5 M$_\odot$ model (100 Myr)



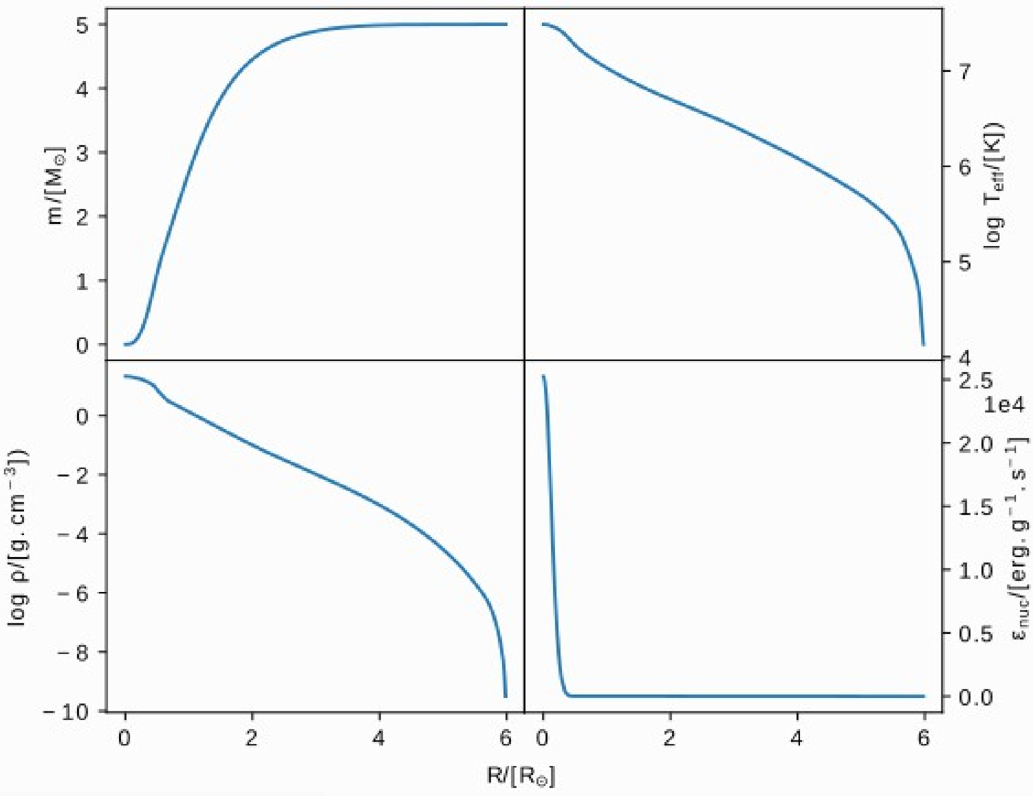
Equilibrium stellar structure

Modules for Experiments in Stellar Astrophisics (MESA)

- **1 $M_\odot$ model (4.5 Gyr)**

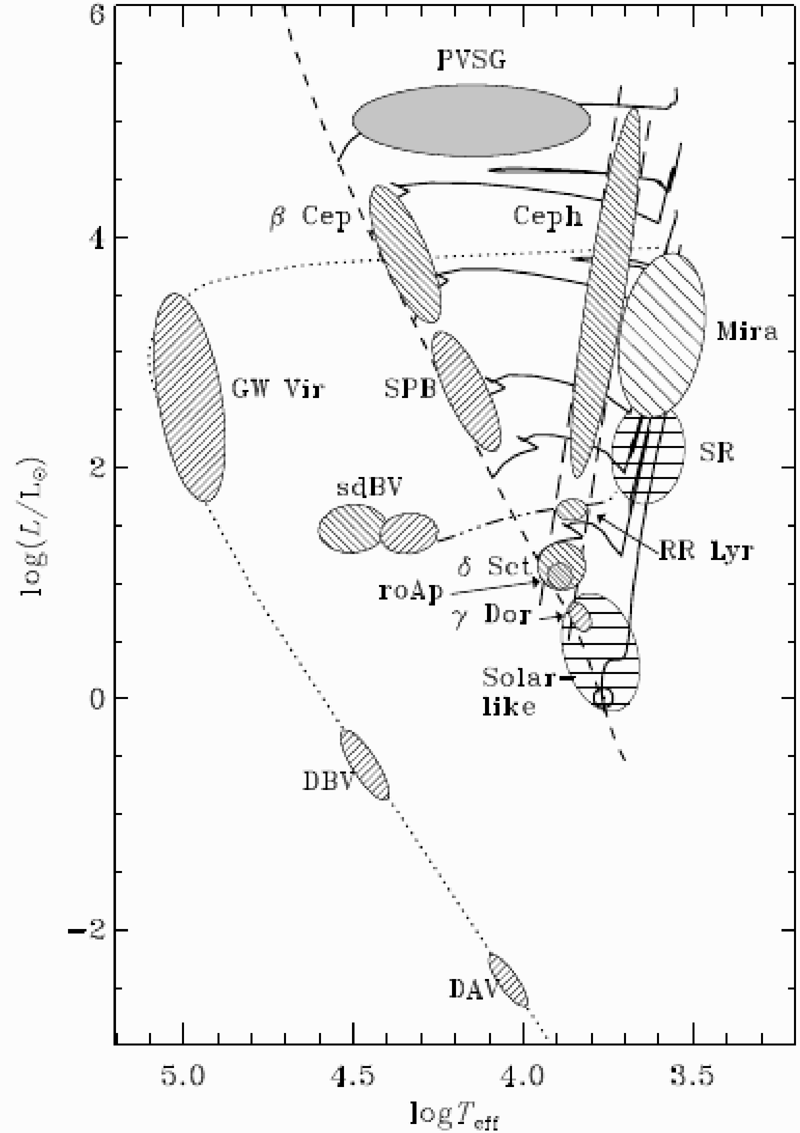


- **5 $M_\odot$ model (100 Myr)**



Pulsations across the HR diagram

- **Periodic change in brightness.**
- **Change in luminosity due to internal processes.**
- **Multiple distinct groups distributed across the main sequence and the instability strip.**



Source: Aerts et al. (2010)

Stellar oscillations

- **Equilibrium disturbed by excitation mechanisms (κ-mechanism, convective blocking, tidal forces).**
- **Governed by hydrodynamic equations.**
- **Stellar matter modelled as non-viscous fluid.**

$$\frac{\partial \rho}{\partial t} + \nabla_{\boldsymbol{r}}(\rho \boldsymbol{u}) = 0$$

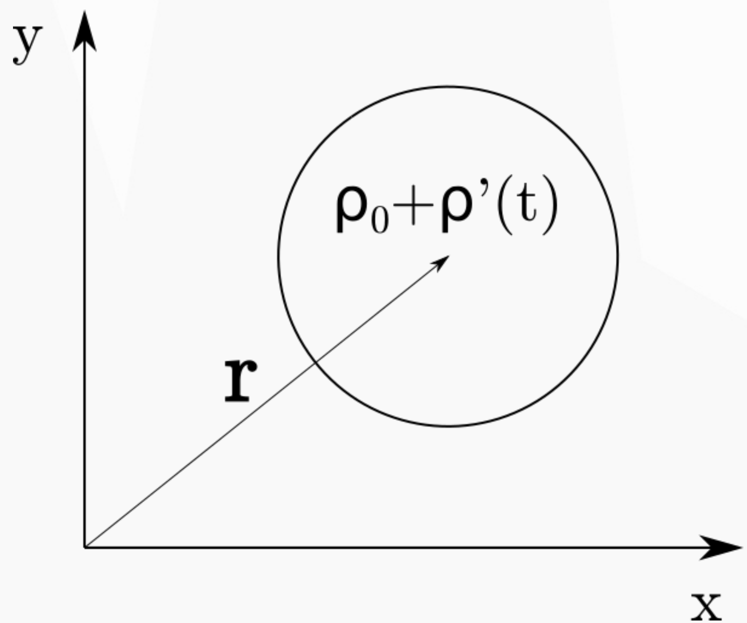
$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla_{\boldsymbol{r}}\right) \boldsymbol{u} = -\nabla_{\boldsymbol{r}} \Psi - \frac{1}{\rho} \nabla_{\boldsymbol{r}} P$$

$$\frac{dq}{dt} = \frac{1}{\rho(\Gamma_3 - 1)} \left(\frac{dP}{dt} - \frac{\Gamma_1 p}{\rho} \frac{d\rho}{dt}\right)$$

Stellar oscillations – perturbative approach

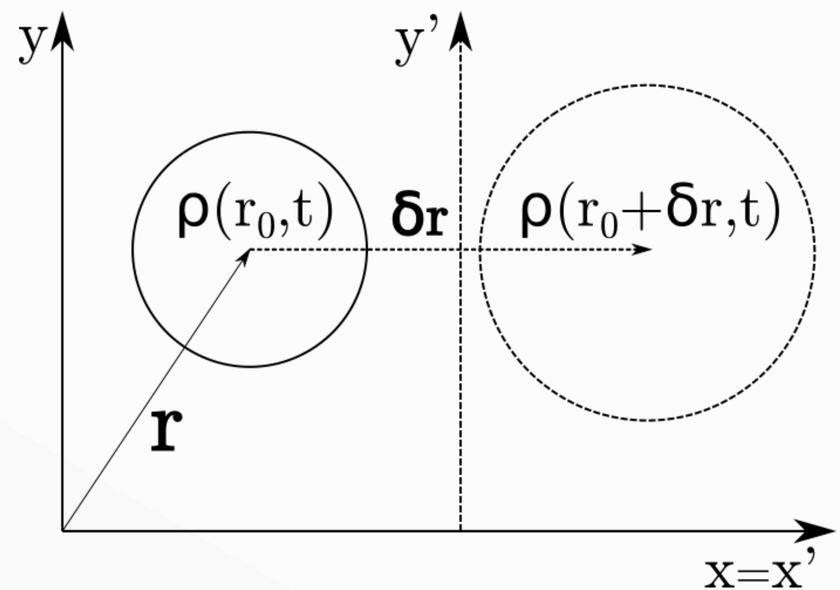
- **Oscillations treated as perturbation of equilibrium stellar structure.**
- **Eulerian perturbation**

$$\rho'(\boldsymbol{r},t) = \rho(\boldsymbol{r},t) - \rho_0(\boldsymbol{r})$$



- **Lagrangian perturbation**

$$\delta\rho(\boldsymbol{r},t) = \rho'(\boldsymbol{r}_0,t) + \boldsymbol{\delta r}\cdot\nabla_{\boldsymbol{r}}\rho_0(\boldsymbol{r}_0).$$



Stellar oscillations – perturbative approach

- **Perturbative approach:**
  - **Decomposition of stellar parameters to equilibrium and perturbed part.**
  - **Subtraction of equilibrium part.**
  - **Linearization.**

$$\rho' + \nabla_{\boldsymbol{r}} \cdot (\rho_0 \boldsymbol{\delta r}) = 0$$

$$\rho_0 \frac{\partial^2 \boldsymbol{\delta r}}{\partial t^2} = -\rho_0 \nabla_{\boldsymbol{r}} \Psi' - \rho' \nabla_{\boldsymbol{r}} \Psi_0 - \nabla_{\boldsymbol{r}} P'$$

$$P' + \nabla_{\boldsymbol{r}} P_0 \cdot \boldsymbol{\delta r} = \frac{\Gamma_{1,0} P_0}{\rho_0} (\rho' + \nabla_{\boldsymbol{r}} \rho_0 \cdot \boldsymbol{\delta r})$$

$$P' = \frac{k_B}{\mu} (\rho_0 T' + T_0 \rho')$$

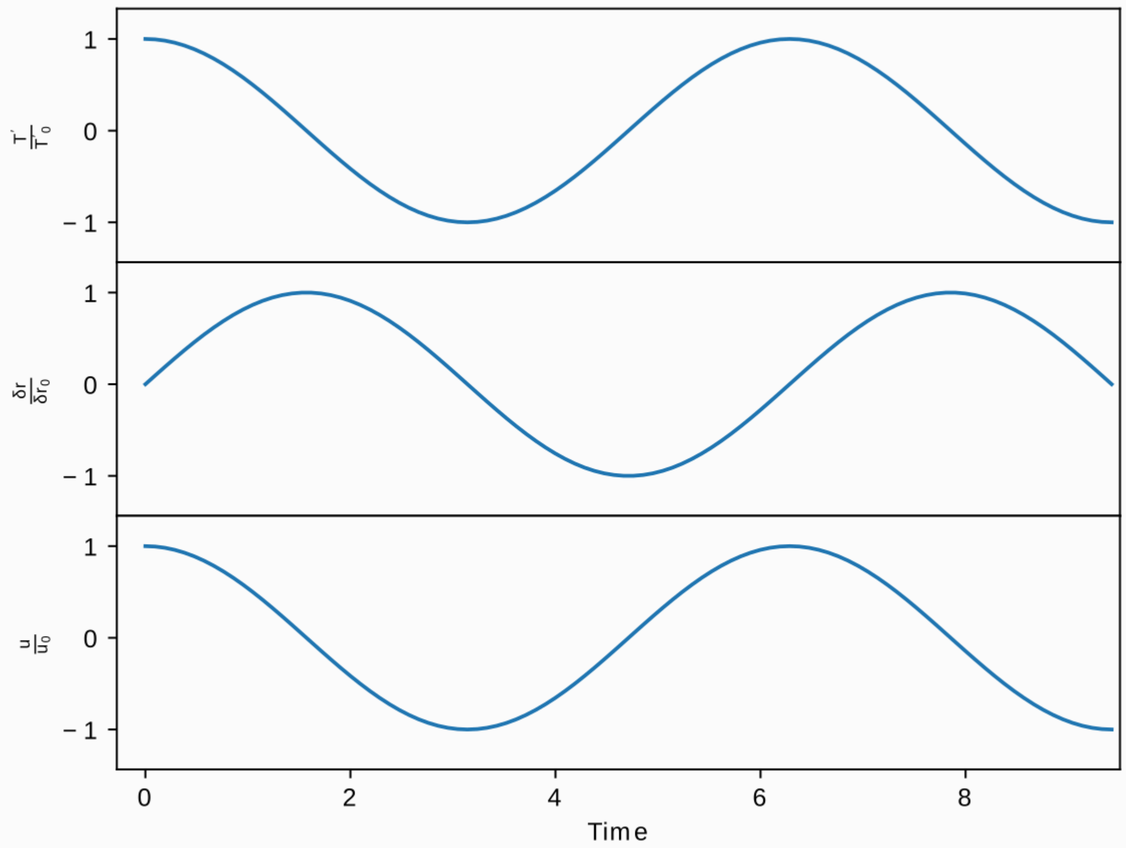
$$\Delta \Psi' = 4\pi G \rho'$$

Pressure driven oscillations

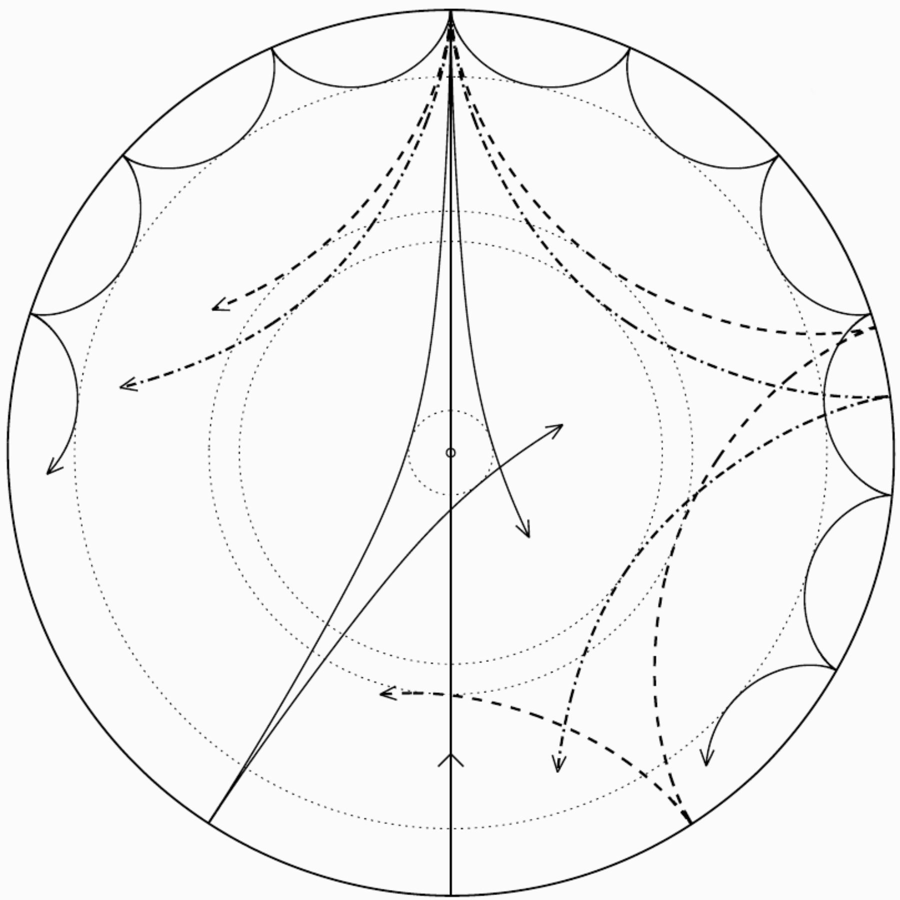
- **Acoustic waves**
- **Local approach – planar wave**

$$\frac{\partial^2 P'}{\partial t^2} = \frac{\Gamma_{1,0} P_0}{\rho_0} \Delta P'$$

$$c_0 = \sqrt{\frac{\Gamma_{1,0} P_0}{\rho_0}}$$

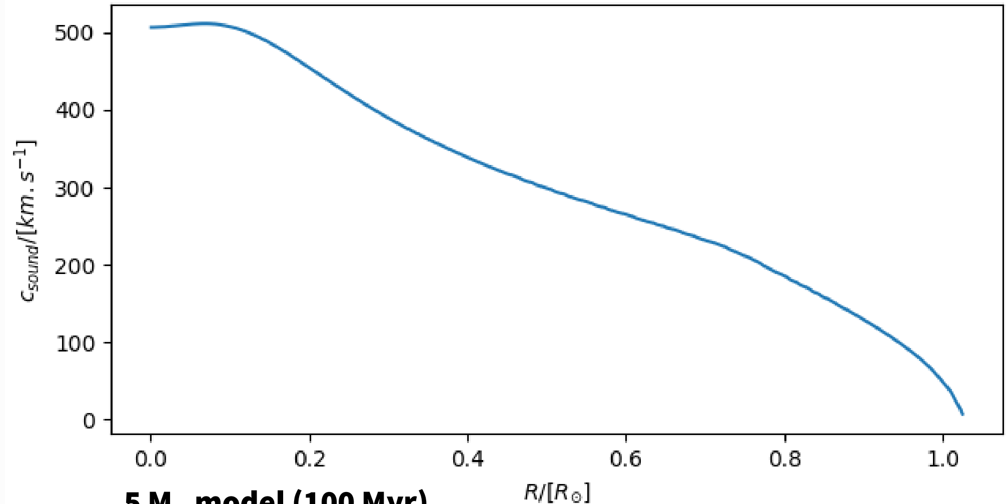


Pressure driven oscillations



Source: Aerts et al. (2010)

1 $M_\odot$ model (4.5 Gyr)



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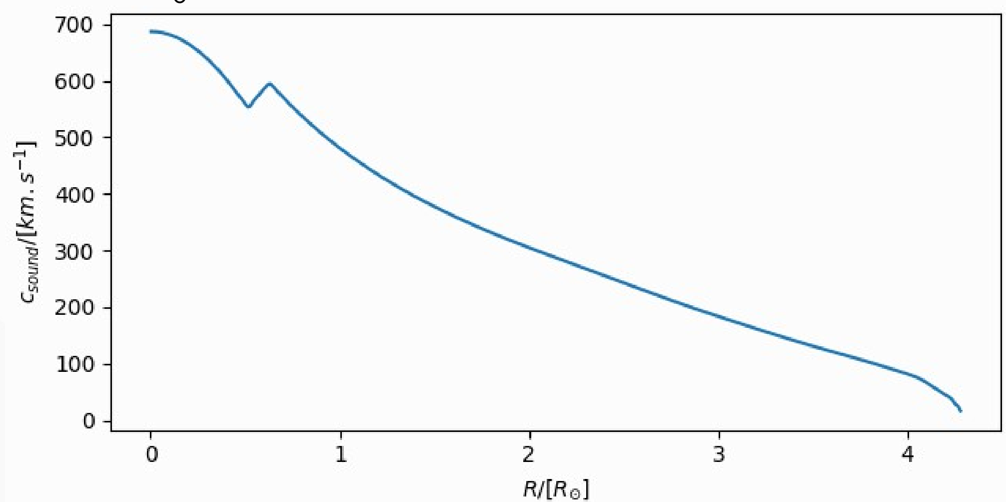
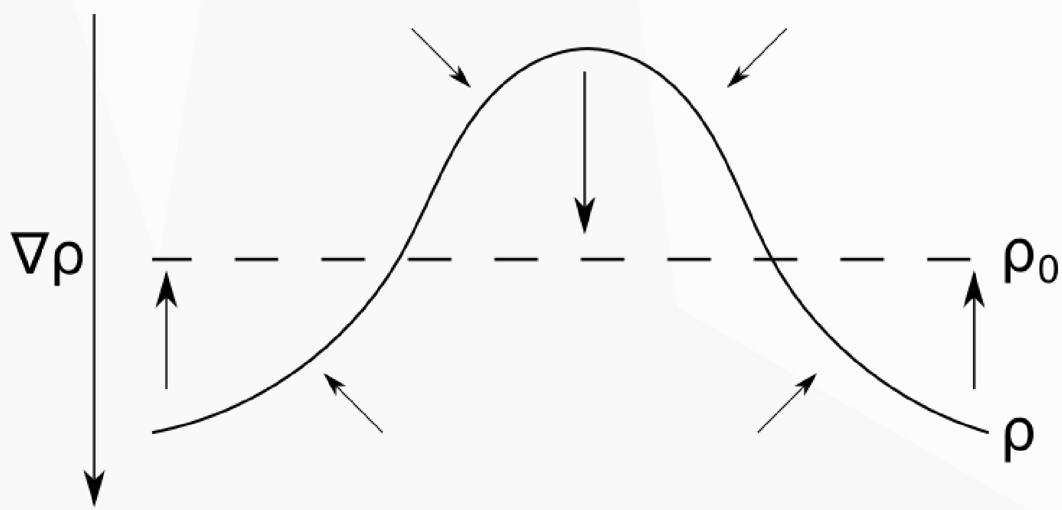
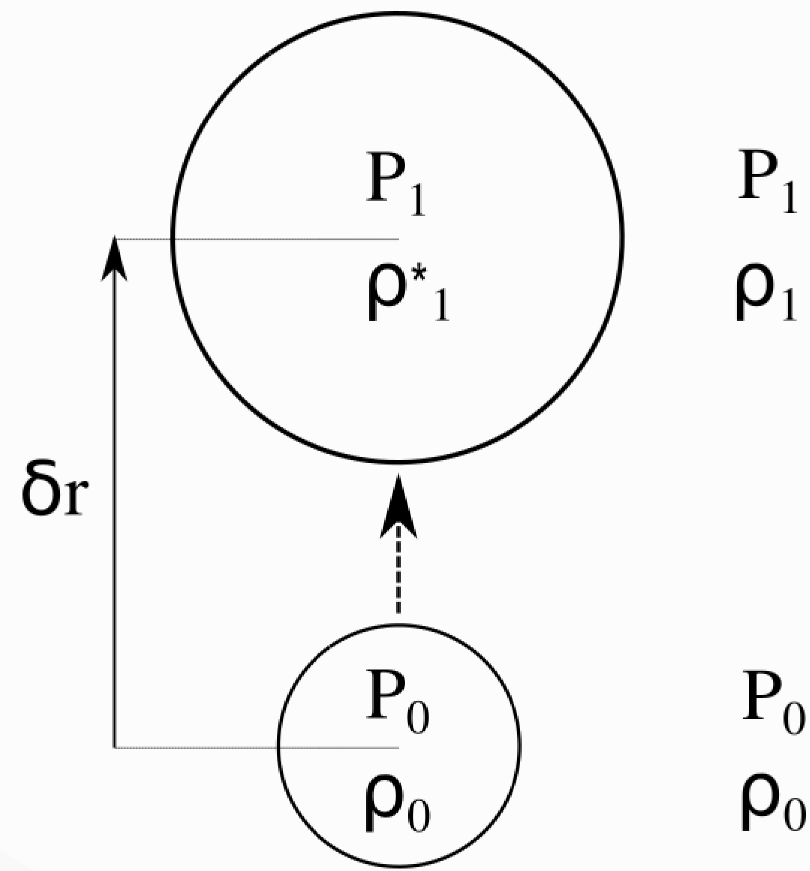


Gravity driven oscillations

$$N^2 = \left(\frac{1}{\Gamma_1}\frac{d\ln P_0}{dr} - \frac{d\ln \rho_0}{dr}\right) g_0 > 0$$

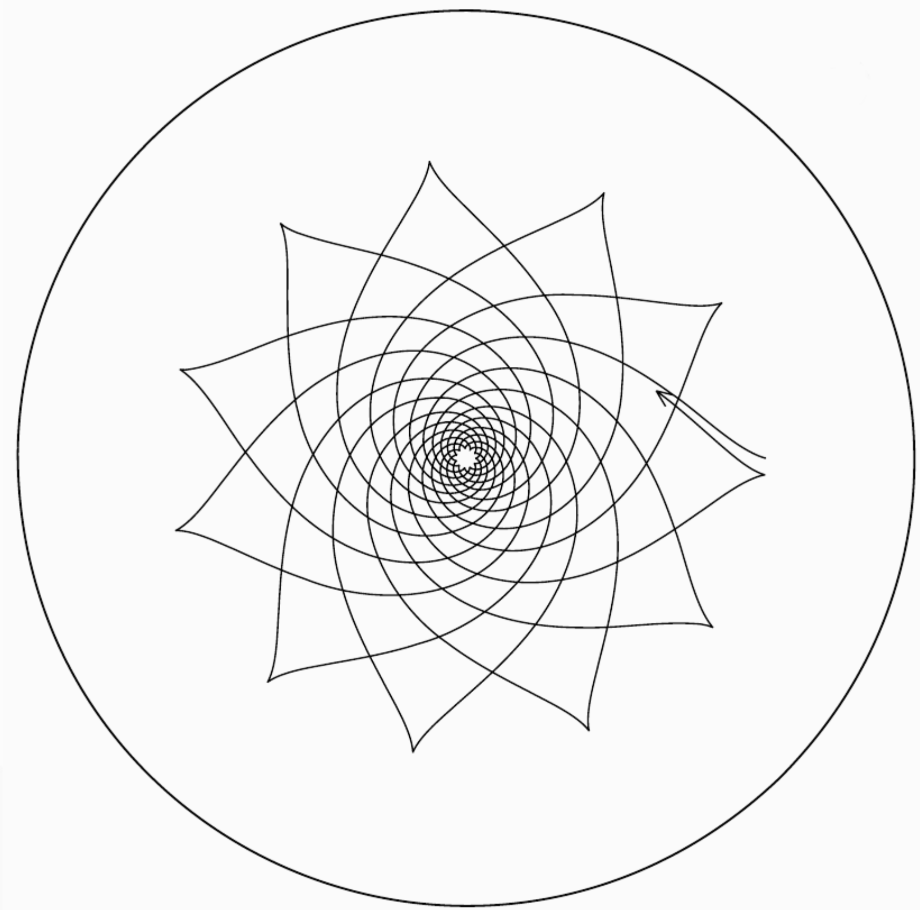




Gravity driven oscillations

- **Dispersion relation**

$$\omega^2 = \frac{N^2}{1 + \left(\frac{k_r}{k_h}\right)^2}$$



Source: Aerts et al. (2010)

Surface representation of oscillations

- **Surface modes – spherical harmonics**

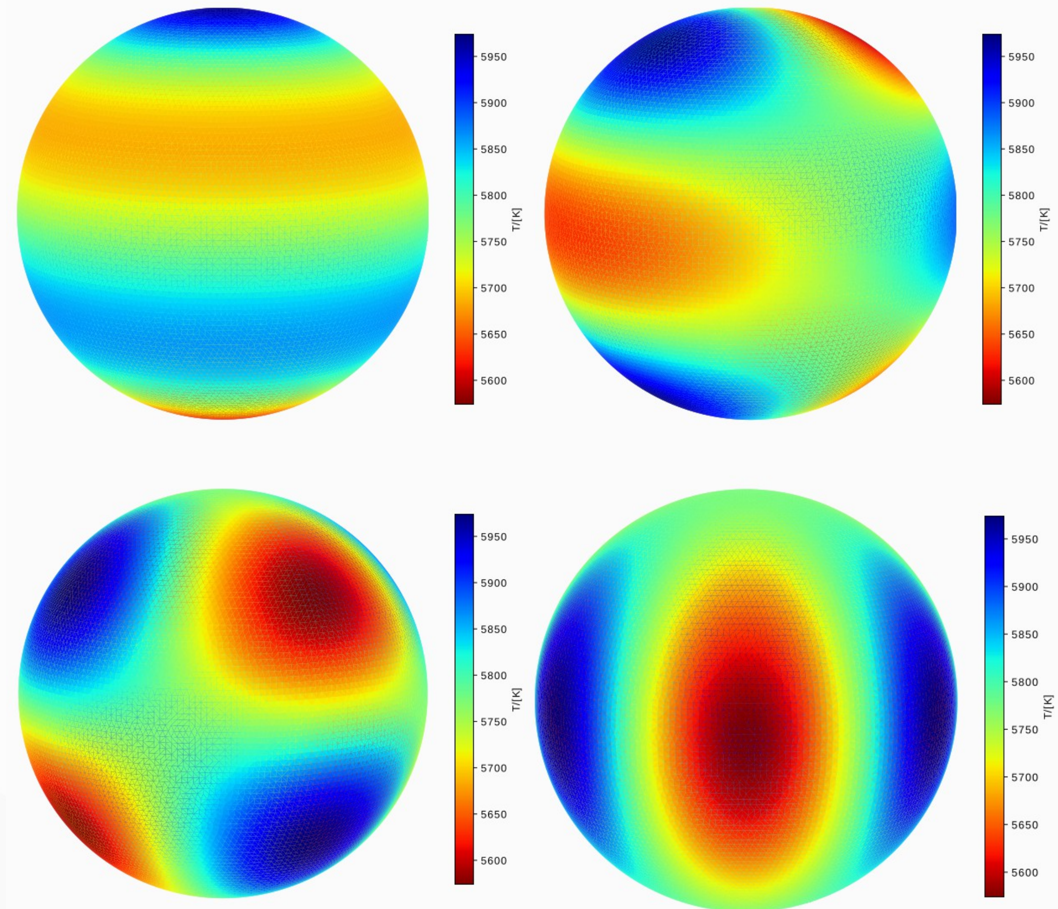
$$Y_l^m(\theta,\varphi) = (-1)^m c_{lm} P_l^m(\cos\theta) e^{im\varphi}$$

- ***l* – no. of surface nodal lines**
- ***m* – no. of azimuthal nodal lines, |*m*| < *l***

$$\xi_r = \sqrt{4\pi}\tilde{\xi}_r(r) Y_l^m(\theta,\phi)\exp(-i\omega t),$$

$$\text{T'} = \sqrt{4\pi}\tilde{T}'(r) Y_l^m(\theta,\varphi)\exp(-i\omega t)$$

$$\tilde{T}'(r) = \frac{\tilde{P}'(r)\mu}{\rho_0 k_B}\left(\frac{\Gamma_{1,0}-1}{\Gamma_{1,0}}\right)$$



Future goals of the project

- **Developing a code able to generate a light curves of eclipsing binaries with a surface inhomogenities.**
- **Explore the solutions of stellar oscillations for heavily distorted components.**
- **Acceleration of computations utilizing GPU computing.**
- **Adapting 1D stellar models to model interiors of deformed stellar components.**

Thank you!
(Any questions?)